POTENTIAL MINIMIZATION IN LEFT-RIGHT SYMMETRIC MODELS

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January 1994

Abstract

We study the Higgs potentials in the Left-Right symmetric model for various choices of Higgs fields. We give emphasis to the cases when the Higgs field $\xi = (2, 2, 15)$ is included to give the correct relations of quark masses and a singlet field η which breaks the left-right parity. As special cases we also include $\xi' = (2, 2, 15)$ and $\chi = (2, 2, 6)$ (which are interesting in the context of the three lepton decay mode of the proton) and field $\delta = (3, 3, 0)$ none of which acquire *vev*. We show that the linear couplings of these fields upon minimization put fine tuning conditions on the parameters of the model. We carry out the minimization of these potentials explicitly. In all the cases the relationship between the *vev*s of the left and right handed triplets v_L and v_R are given. The phenomenological consequences of this minimization regarding the neutrino masses are also studied.

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1   INTRODUCTION

Left-Right symmetric models[1] are considered to be the most natural extensions of the standard model. Popularly one chooses the gauge group $G_{3221} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ or $G_{224} = SU(2)_L \times SU(2)_R \times SU(4)_c$ to describe the invariance properties of the model. When G_{3221} or G_{224} admits spontaneous symmetry breaking one recovers the standard model. Spontaneous symmetry breakdown takes place when the Higgs fields transforming nontrivially under the higher symmetry group but not transforming under the lower symmetry group acquires a vacuum expectation value (*vev*). If one embeds the group G_{3221} or G_{224} in a grand unified theory or a partially unified theory then LEP constraints on $sin^2\theta_w$[2] can put strong bounds on the breaking scale of the right handed $SU(2)_R$ group. On the other hand if one considers the left-right symmetric model with $g_L \neq g_R$ the right handed breaking scale can be lowered[3]. In this case the model becomes interesting as a rich set of phenomenological consequences can be directly tested in the next generation colliders. To achieve the inequality of the couplings a D-odd singlet Higgs field η is introduced which on acquiring vev breaks the left-right parity (D-parity).

It is well known that the coupling of the Higgs field $\phi = (2, 2, 1)$ under G_{224} gives the masses to fermions in the left right model. It is also known that the *vev* of ϕ alone cannot generate the correct relationship of the quark and the lepton masses. One has to introduce a field $\xi = (2, 2, 15)$ to genetate the

correct mass relations[4]. We have included the field ξ in our Higgs potential in the presence and in the absence of D-parity breaking. We have shown how the see-saw relationship gets modified in the presence of the *vev* of the field ξ.

In recent past much interest is generated in the three lepton decay mode of proton in left right symmetric model. In this scenario one introduces the fields $\xi' = (2, 2, 15)$ or $\chi = (2, 2, 6)$. Here due to the mixing of this new fields with the field ξ the $SU(3)_c$ triplet component of ξ remains light which in turn mediate the three lepton decay of the proton with a lifetime of 4×10^{31} years[5]. It is argued that this decay channel can produce electron type neutrino and which may lead to an explanation of the atmospheric neutrino problem. We include this extra fields in special cases of our analysis. We show that the linear couplings of these extra fields with the other scalars present in the model can put constraints on the parameters of the model and the *vev*s of the scalar fields.

Next we consider the introduction of the scalar $\delta = (3, 3, 0)$ which does not acquire *vev* in the model. We show that the linear coupling of δ with other fields in the model puts constraints on the parameters of the model. In particular we show that in the presense of δ the scale of D-parity breaking has to be very close to the right handed breaking scale.

In all the cases described above we perform the minimization of the scalar potential in detail and write the relationships among the *vev* s of the left

handed and right handed triplets. In most cases the *vev* of the left handed triplet has to be nonzero and it contributes to the neutrino mass[1]. The see-saw mass becomes comparable to this term. The see-saw mass becomes compapable to this term after severely fine tuning some parameters. This fine tuning can be avoided if D-parity is broken. We investigate in this paper whether such features are maintained in the presence of ξ.

This paper is structured as the following. In section 2 we describe the basics of the left right model and summarize the Higgs choices of the model. In section 3 we summarize the results available in the literature and perform the potential minimization in the presence of the field ξ. In section 4 we introduce the fields ξ', χ, and δ. We show that the linear couplings of these fields put strong constraints on the model parameters. In section 5 we comment on neutrino mass matrices. In section 6 we state the conclusions.

2 RUDIMENTS OF LEFT RIGHT SYMMETRIC MODEL

In this paper we are interested in the following symmetry breaking pattern;

$$
\begin{aligned}
SU(2)_L \times SU(2)_R \times SU(4)_c \quad &\xrightarrow{M_X} \quad SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
&\xrightarrow{M_R} \quad SU(2)_L \times SU(3)_c \times U(1)_Y \\
&\xrightarrow{M_W} \quad SU(3)_c \times U(1)_Q
\end{aligned}
\tag{1}
$$

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[1]However in the presence of both ξ and χ one has to have also the presence of a D-odd singlet η to predict an acceptable relationship between v_L and v_R

If G_{224} is embedded in any higher symmetry group, then also most of the analysis will not change. In this sense our analysis is quite general. The advantage of starting with the group G_{224} instead of the group G_{3221} is that, we can discriminate between the fields which do and don't distinguish between quarks and leptons. This is important to understand the mass ratios of quarks and leptons.

We will also assume that $M_X = M_R$ which will imply that the scale of breaking of SU(4) color is the same as that of the breaking of the left right symmetry. This will not cause any loss of generality of our analysis. To specify the model further let us state the transformation properties of the fermions.

$$
\begin{aligned}
\psi_L &= \begin{pmatrix} \nu_L \\ e^-{}_L \end{pmatrix} : (2,1,4) \quad ; \quad \psi_R = \begin{pmatrix} \nu_R \\ e^-{}_R \end{pmatrix} : (2,1,4) \\
Q_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix} : (2,1,4) \quad ; \quad Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} : (2,1,4)
\end{aligned}
\tag{2}
$$

The scalar fields which may acquire *vev* are stated below.

$$
\phi_1 \equiv (2,2,1) \quad ; \quad \phi_2 \equiv \tau_2 \phi_1^* \tau_2 \quad ; \quad \xi_1 \equiv (2,2,15) \quad ; \quad \xi_2 \equiv \tau_2 \xi_1^* \tau_2
$$

$$
\Delta_L \equiv (3,1,10) \quad ; \quad \Delta_R \equiv (1,3,10) \quad , \quad \eta \equiv (1,1,0)
$$

It has been shown in recent past that the LEP constraints on $sin^2\theta_w$[2] can put strong lower bound on the scale M_R. From renormalization group equations one can show that the right handed breaking scale has to be greater than 10^9 GeV. However one can show that when the D-Parity is broken

the right handed breaking scale can be lowered. In that case a rich set of phenomenological predictions can be experimentally tested in high energy colliders. Here we consider the singlet field η which is odd under D-Parity. It breaks D-Parity when it acquires *vev*[3].

If we consider an underlying GUT, and start with the masses of the quarks and leptons to be the same at the unification scale, then, in the absence of ξ the low energy mass relations of fermions are not correct. This is because the field (2,2,1) contributes equally to the masses of the quarks and leptons. The situation can be corrected by the introduction of the field (2,2,15)[4]. This is the initial motivation to introduce the field ξ. Once it is there it allowes new interesting baryon number violating decay modes which we discuss below.

Recently a lot of interest has been generated in the three lepton decay of the proton in SU(4) color gauge theory[5]. It can be shown that if the SU(3) triplet component of ξ remains sufficiently light it can mediate the three lepton decay mode of proton with a lifetime of 4×10^{31} years. In that case sufficient number of extra electron type neutrinos can be produced in the detector which can explain atmospheric neutrino anomaly. To keep the SU(3) triplet component of ξ sufficiently light, the following mechanism was proposed by Pati, Salam and Sarkar. If an extra (2,2,15) or (2,2,6) Higgs field (henceforth called ξ' and χ) is introduced, its SU(3) triplet component will mix with the triplet component of ξ and hence there will be a light triplet in the model. These extra fields does not acquire *vev*. However the terms in the scalar potential which are linear in these extra fields can strongly constrain

the other parameters of the model. In this paper we introduce such extra fields which does not acquire *vev* and study the terms in the scalar potential which are linear in these extra fields. The extra fields we consider here are,

$$\xi' = (2, 2, 15) \; ; \; \chi = (2, 2, 6) \; ; \; \delta = (3, 3, 0) \tag{3}$$

We shall see below that the linear term in the extra field δ will constrain the ratio of the D-parity breaking scale and the right handed symmetry breaking scale. We emphasize that in different models with extra scalars such study is necessary as it points out the extra scalar which is not favorable by the existing phenomenology.

3 MINIMIZATION OF POTENTIAL

3.1 MINIMAL CHOICE OF HIGGS SCALARS

The general procedure we adopt here is the following. First we write down the most general scalar potential which is allowed by renormalizability and gauge invariance. Next we substitute the vacuum expectation values in the potential and find out the minimization conditions. Here let us first write down the scalar potential with the scalar fields ϕ, Δ and $\eta[6]$,

$$V(\phi_1, \phi_2, \Delta_L, \Delta_R, \eta) = V_\phi + V_\Delta + V_\eta + V_{\eta\Delta} + V_{\eta\Delta} + V_{\eta\phi} \tag{4}$$

Where the different terms in this expression are given by,

$$V_\phi = -\sum_{i,j} \mu_{ij}^2 \ tr(\phi_i^\dagger \phi_j) + \sum_{i,j,k,l} \lambda_{ijkl} \ tr(\phi_i^\dagger \phi_j) \ tr(\phi_k^\dagger \phi_l)$$
$$+ \sum_{i,j,k,l} \lambda_{ijkl} \ tr(\phi_i^\dagger \phi_j \phi_k^\dagger \phi_l)$$

$$V_\Delta = -\mu^2 \ (\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R) + \rho_1 \ [tr(\Delta_L^\dagger \Delta_L)^2 + tr(\Delta_R^\dagger \Delta_R)^2]$$

$$+ \rho_2 \ [tr(\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L) + tr(\Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R)] + \rho_3 \ tr(\Delta_L^\dagger \Delta_L \Delta_R^\dagger \Delta_R)$$

$$V_\eta = -\mu_\eta^2 \ \eta^2 + \beta_1 \ \eta^4$$

$$V_{\Delta\phi} = +\sum_{i,j} \alpha_{ij} \ (\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R) \ tr(\phi_i^\dagger \phi_j) + \sum_{i,j} \beta_{ij} \ [\ tr(\Delta_L^\dagger \Delta_L \phi_i \phi_j^\dagger)$$
$$+ tr(\Delta_R^\dagger \Delta_R \phi_i^\dagger \phi_j)]$$
$$+ \sum_{i,j} \gamma_{ij} \ tr(\Delta_L^\dagger \phi_i \Delta_R \phi_j^\dagger)$$

$$V_{\eta\Delta} = M \ \eta \ (\Delta_L^\dagger \Delta_L - \Delta_R^\dagger \Delta_R) + \beta_2 \ \eta^2 \ (\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R)$$

$$V_{\eta\phi} = \sum_{i,j} \delta_{ij} \ \eta^2 \ tr(\phi_i^\dagger \phi_j)$$

The vacuum expectation values of the fields have the following form:

$$< \phi > \; = \; \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \quad ; \quad < \Delta_L > = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix} \quad ; \quad < \eta > = \eta_0;$$

$$< \tilde{\phi} > \; = \; \begin{pmatrix} k' & 0 \\ 0 & k \end{pmatrix} \quad ; \quad < \Delta_R > = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

The phenomenological consistency requires the hierarchy $< \Delta_R > \; >> \; < \phi > \; >> \; < \Delta_L >$ and also that $k' << k$. Now the minimization conditions of the potential V is found by differentiating it with respect to the parameters k, k', v_L, v_R and η_0 and separately equating them to zero. This will give us five equations for five parameters present. Solving the equations involving the derivatives with respect to v_L and v_R we get the relation between v_L and v_R. The details of the derivation is presented in the appendix.

$$v_L v_R \; = \; \frac{\beta k^2}{[(\rho - \rho') + \frac{4M\eta_0}{(v_L^2 - v_R^2)}]}$$

Here we have defined $\beta = 2\gamma_{12}$. We get in the M=0 limit,

$$v_L v_R \simeq \frac{\beta k^2}{[\rho - \rho']} \simeq \gamma k^2 \tag{5}$$

Here γ is a function of the couplings. However when the field η is present, v_L becomes differently related to v_R in the limit of large η_0.

$$v_L \simeq -(\frac{\beta k^2}{4M\eta_0})v_R \simeq (\frac{\beta k^2}{\eta_0^2})v_R \tag{6}$$

Here we see the important difference between the D-conserving and D-breaking scenarios.

This result was discussed in details in ref. [7]. In the D-parity conserving case, when the η field is absent one has to fine tune parameters to make γ arbitrarily snall so that the see-saw neutrino mass can be comparable to the Majorana mass of the left-handed neutrinos given by v_L. This fine tuning becomes redundant when the field η acquires vev.

3.2   IN THE PRESENCE $\xi=(2,2,15)$

When ξ is present the most general scalar potential takes the following form:

$$V(\phi_1, \phi_2, \Delta_L, \Delta_R, \xi_1, \xi_2, \eta) = V_\phi + V_\Delta + V_\eta + V_\xi + V_{\phi\eta} + V_{\eta\Delta} + V_{\eta\phi} + V_{\phi\xi} + V_{\Delta\xi} + V_{\eta\xi} \tag{7}$$

The explicit forms of the terms involving ξ are listed below,

$$V_\xi = -\sum_{i,j} m_{ij}^2 \; tr(\xi_i^\dagger \xi_j) + \sum_{i,j,k,l} n_{ijkl} \; tr(\xi_i^\dagger \xi_j \xi_k^\dagger \xi_l) + \sum_{i,j,k,l} p_{ijkl} \; tr(\xi_i^\dagger \xi_j) \; tr(\xi_k^\dagger \xi_l)$$

$$V_{\phi\xi} = \sum_{i,j,k,l} u_{ijkl} \; tr(\phi_i^\dagger \phi_j \xi_k^\dagger \xi_l) + \sum_{i,j,k,l} v_{ijkl} \; tr(\phi_i^\dagger \phi_j) \; tr(\xi_k^\dagger \xi_l)$$

$$\begin{aligned} V_{\Delta\xi} = \; & +\sum_{i,j} a_{ij} \; [\; tr(\Delta_L^\dagger \Delta_L) + tr(\Delta_R^\dagger \Delta_R)] \; tr(\xi_i^\dagger \xi_j) \\ & +\sum_{i,j} b_{ij} \; [\; tr(\Delta_L^\dagger \Delta_L \xi_i \xi_j^\dagger) + tr(\Delta_R^\dagger \Delta_R \xi_i^\dagger \xi_j)] \\ & +\sum_{i,j} c_{ij} \; tr(\Delta_L^\dagger \xi_i \Delta_R \xi_j^\dagger) \end{aligned}$$

$$V_{\eta\xi} \;=\; \sum_{i,j} d_{ij}\;\eta^2\;tr(\xi_i^\dagger \xi_j)$$

The vacuum expectation value of ξ has the following form,

$$<\xi> = \begin{pmatrix} \tilde{k} & 0 \\ 0 & \tilde{k}' \end{pmatrix} \times (1,1,1,-3) \tag{8}$$

Here we may briefly mention the need to introduce the field ξ. The vacuum expectation value of the field ϕ is given by,

$$<\phi> = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \times (1,1,1,1) \tag{9}$$

Note that in the SU(4) color space the fourth entry is 1 for the *vev* of ϕ whereas it is -3 for the *vev* of ξ. Hence the *vev* of ϕ treats the quarks and the leptons on the same footing, whereas the *vev* of ξ differentiates between the quarks and the leptons. For example in the absence of ξ one gets $m_e^0 = m_d^0$, $m_\mu^0 = m_s^0$ and $m_\tau^0 = m_b^0$. Now including the QCD and electroweak renormalization effects in the symmetric limit it leads to the relation $\frac{m_e}{m_\mu} = \frac{m_d}{m_s}$. However when the field ξ is included in the masses in the symmetric limit they take the form $m_e^0 = m_e^\phi - 3m_e^\xi$ and $m_d^0 = m_d^\phi - m_d^\xi$.

The minimization condition is again found by taking the derivatives of V with respect to $k, k', v_L, v_R, \tilde{k}^2$ and $\tilde{k}'^2$ and separately equating them to zero. Solving the equations involving the derivatives of v_L and v_R yields in the limit $\tilde{k}' << \tilde{k}$:

$$v_L v_R \;=\; \frac{[(w\tilde{k}^2 + \beta k^2)\,(v_L^2 - v_R^2)}{[(\rho - \rho')(v_L^2 - v_R^2) + 4M\eta_0]} \tag{10}$$

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Here we have defined $w = 2c_{12}$. Let us again check the special cases. Firstly the case without ξ can be recovered in the limit w=0, on the other hand the case with unbroken D-parity can be restored in the limit M=0. Which is,

$$v_L v_R \simeq \frac{w\tilde{k}^2 + \beta k^2}{[(\rho - \rho')]} \tag{11}$$

When D-parity is broken the v_L can be suppressed by η_0,

$$v_L = \frac{w\tilde{k}^2 + \beta k^2}{\eta_0^2} v_R \tag{12}$$

We infer that the field ξ is allowed by the potential minimization and its introduction does not alter the general features of the see-saw condition between v_L and v_R.

4 INTRODUCTION OF EXTRA FIELDS

4.1 INTRODUCTION OF ξ'=(2,2,15)

We have already mentioned that there exists interesting models in the literature where the field ξ' is introduced to induce a sufficiently large amplitude of the three lepton decay width of the proton. In these models the field ξ does not acquire *vev*. Hence after the minimization all terms other than the ones which are linear in ξ' drops out whereas the ones which are linear in ξ' puts constraints on the other parameters of the model. Usually when any new fields are introduced in any model, which don't acquire *vev*s, it is assumed that it will not change the minimization conditions. As a result potential minimization with such fields were not done so far.

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In this section we will first write down what are the linear couplings of the field ξ'.

$$
\begin{aligned}
V'_\xi \;=\; & -\sum_{i,j}\tilde{m}^2_{ij}\; tr(\xi^\dagger_i\xi'_j) + \sum_{i,j,k,l} n_{ijkl}\; tr(\xi^\dagger_i\xi_j\xi^\dagger_k\xi'_l) \\
& + \sum_{i,j,k,l} p_{ijkl}\; tr(\xi^\dagger_i\xi_j)\; tr(\xi^\dagger_k\xi'_l) \\[2ex]
& + \sum_{i,j,k,l} u_{ijkl}\; tr(\phi^\dagger_i\phi_j\xi^\dagger_k\xi'_l) + \sum_{i,j,k,l} v_{ijkl}\; tr(\phi^\dagger_i\phi_j)\; tr(\xi^\dagger_k\xi'_l) \\[2ex]
& + \sum_{i,j}\tilde{a}_{ij}\;(\Delta^\dagger_L\Delta_L + \Delta^\dagger_R\Delta_R)\; tr(\xi^\dagger_i\xi'_j) \\
& + \sum_{i,j}\tilde{b}_{ij}\;[\; tr(\Delta^\dagger_L\Delta_L\xi'_i\xi^\dagger_j) + tr(\Delta^\dagger_R\Delta_R\xi^\dagger_i\xi'_j)] \\
& + \sum_{i,j}\tilde{c}_{ij}\; tr(\Delta^\dagger_L\xi'_i\Delta_R\xi^\dagger_j) \\[2ex]
& + \sum_{i,j}\tilde{d}_{ij}\;\eta^2\; tr(\xi^\dagger_i\xi'_j)
\end{aligned}
$$

When this potential is minimized with respect to ξ' we get a relation between the couplings and the *vev* s. Obviously in this case due to large number of couplings of the field ξ' (which are independent parameters) this condition can be easily satisfied. A more stringer and interesting situation is the case where an extra field χ is introduced instead of ξ'.

4.2 INTRODUCTION OF $\chi=(2,2,6)$

It has been pointed out by Pati[8] that the field χ is a very economical choice for the mechanism that leads to appriciable three lepton decay of proton. The field χ is contained in the field 54-plet of $SO(10)$ which has to be present for the breaking of $SO(10)$. The terms linear in χ can be written as:

$$V_\chi = P \; \eta\xi\chi(\Delta_R - \Delta_L) + M \; \chi\xi(\Delta_R + \Delta_L) \tag{13}$$

These terms upon minimization gives the condition

$$v_L = \frac{P\eta_0 - M}{P\eta_0 + M}v_R \tag{14}$$

This means that to get $v_R >> v_L$ one has to fine tune $P\eta_0 - M << P\eta_0 + M$. This is interesting in the context of the three lepton decay of Proton which will be discussed elsewhere [9].

4.3 INTRODUCTION OF $\delta=(3,3,0)$

In this case we first write down the linear couplings of the field δ:

$$V_\delta = M_1\delta(\Delta_L\Delta_R^\dagger + \Delta_R\Delta_L^\dagger) + M_2\delta\phi\phi^\dagger + C_1\eta\delta(\Delta_L\Delta_R^\dagger + \Delta_R\Delta_L^\dagger) + C_2\eta\delta\phi^\dagger\phi \tag{15}$$

These terms upon minimization gives the following conditions,

$$v_L v_R = -\frac{M_2 + C_2\eta_0}{2M_1 + C_1\eta_0}k^2 \tag{16}$$

In the limit of very large η_0 we can write,

$$v_L v_R \simeq k^2 \tag{17}$$

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If we compare this relation with the see-saw relation of section 3.2 we get,

$$\frac{v_R^2}{\eta_0^2} = \frac{k^2}{w\tilde{k}^2 + \beta k^2} \simeq O(1) \tag{18}$$

Thus due to the introduction of δ the left-right parity and the left right symmetry gets broken almost at the same scale.

5   NEUTRINO MASS MATRIX

The fermions acqire masses through the Yukawa terms in the lagrangian when the Higgs fields acquire vev. The Yukawa part in the Lagrangian written in terms of fermionic and Higgs fields is given by,

$$
\begin{aligned}
L_{Yukawa} &= y_1(\bar{f}_L f_R \phi_1) + y_2(\bar{f}_L f_R \phi_2) + y_3(\bar{f}_L^c f_L \Delta_L + \bar{f}_R^c f_R \Delta_R) \\
&\quad + y_4(\bar{f}_L f_R \xi_1) + y_5(\bar{f}_L f_R \xi_2)
\end{aligned} \tag{19}
$$

where y_i (i=1,5) are Yukawa couplings. With this notation neutrino mass matrix written in the basis (ν_L, ν_L^c) is

$$M = \begin{pmatrix} m_{M_L} & m_D \\ m_D & m_{M_R} \end{pmatrix} \tag{20}$$

where m_{M_L} (m_{M_R}) is the left (right) handed Majorana mass term whereas m_D is the Dirac mass term. These terms can be related to the Yukawa couplings and *vev*s through the following relation,

$$
\begin{aligned}
m_{M_L} &= y_3 v_L \\
m_D &= (y_1 + y_2)(k + k') + (y_4 + y_5)(\tilde{k} + \tilde{k}') \\
m_{M_R} &= y_3 v_R
\end{aligned} \tag{21}
$$

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Upon diagonalization of the mass matrix we obtain the mass eigenvalues. Now let us consider the simplifying assumption that all the Yukawa couplings are of order "h" and the *vev* s k' and $\tilde{k}'$ are much smaller than the *vev* s k and $\tilde{k}$ respectively. Under this assumption the eigenvalues become,

$$m_1 = y_3 v_R$$

$$m_2 = m_{M_L} - \frac{M_D^2}{m_{M_R}} = y_3 v_L - \frac{h^2(k^2 + \tilde{k}^2)}{y_3 v_R}$$

We substitute for v_L from the see-saw condition to get in the D-parity conserving $g_L = g_R$ case,

$$m_2 = y_3 \frac{(\beta k^2 + w\tilde{k}^2)}{v_R} - \frac{h^2(k^2 + \tilde{k}^2)}{y_3 v_R} \tag{22}$$

We notice that the second term in the right hand side is suppressed by the square of the Yukawa coupling. Due to this the first term dominates. If we want to make the first term small compared to the second we need to fine tune the parameters. Hence one has to fine tune such that $\beta k^2 + w\tilde{k}^2 \simeq 0$ to get acceptable value of the the light neutrino mass. However in the presence of the *vev* of η we get,

$$m_2 = y_3 \frac{w\tilde{k}^2 + \beta k^2}{\eta_0^2} v_R - \frac{h^2(k^2 + \tilde{k}^2)}{y_3 v_R} \tag{23}$$

In the limit of very large η_0 the first term drops out of the expression and one gets rid of the fine tuning problem. However, if the field δ (which does not acquire any *vev*) is present, we cannot get away with the fine tuning problem, since it is difficult to maintain $v_R \ll \eta_0$.

6 CONCLUSIONS

We have incorporated the scalar field $\xi=(2,2,15)$ in the scalar potential of the $SU(4)_{color}$ left-right symmetric extension of the standard model. This field is necessary to predict correct mass relationships of the quarks and the leptons. After including the field ξ in the scalar potential we have carried out the minimization of potential, and worked out the relationship between the *vev*s of the left-handed and the right-handed triplets (see-saw relationship). We have shown that the field ξ is allowed by potential minimization and its inclusion does not change the qualitative nature of the see-saw relationship existing in literature. Once the see-saw relationship between the v_L and v_R is known we have gone ahead to construct the neutrino mass matrix. We have shown that even after the inclusion of the field ξ one needs to fine tune the parameters in the $g_L = g_R$ case to predict small mass for the left handed neutrino, while in the $g_L \neq g_R$ case one naturally gets a large suppression for the left handed neutrino mass. This happens because even after the inclusion of the field ξ the light neutrino mass gets suppressed by the *vev* of the D-odd singlet η rather than the *vev* of Δ_R.

If there are new scalar fields which don't acquire any *vev*, then to check the consistency one has to write down their linear couplings with other fields and after minimizing the potential use the appropriate *vev*s of the various fields. In some cases the presence of such fields can give new interesting phenomenology. We studied some such cases for demonstration.

In recent past it has been shown that the three lepton decay of the proton can successfully explain the atmospheric neutrino anomaly by producing excess of electron type neutrino in the detector. To produce phenomenologically acceptable decay rate in the three lepton decay mode a mechanism was suggested by Pati, Salam and Sarkar, and later by Pati. In this mechanism one has to include extra scalars ξ'=(2,2,15) or χ=(2,2,6) which does not acquire *vev*. We have calculated the linear couplings of such terms in the scalar potential and shown that these terms give relations that constrain the values of parameters and *vev* s of the model. In this paper we have given these constraints. We have also included as a special case the extra scalar δ=(3,3,0) and shown that its inclusion forces the right handed breaking scale and the D-parity breaking scale to become almost equal.

APPENDIX

When the spontaneous symmetry breakdown (SSB) occurs the scalar fields acquire *vev*. Let us first the case when we include only the fields ϕ, Δ_L and Δ_R. The potential after the SSB looks like the following,

$$
\begin{aligned}
V_1 &= -\mu^2 \ (v_L^2 + v_R^2) + \frac{\rho}{4} \ (v_L^4 + v_R^4) + \frac{\rho'}{4} \ (v_L^2 v_R^2) + 2v_L v_R [(\gamma_{11} \\
&\quad + \gamma_{22}) k k' + \gamma_{12}(k^2 + k'^2)] + (v_L^2 + v_R^2) \ [(\alpha_{11} + \alpha_{22} + \beta_{11}) \ k^2 \\
&\quad + (\alpha_{11} + \alpha_{22} + \beta_{22}) \ k'^2 + (4\alpha_{12} + 2\beta_{12}) \ k k'] \\
&\quad + \text{terms containing } k \text{ and } k' \text{ only} \hspace{4cm} (24)
\end{aligned}
$$

We have defined the new parameters as $\rho = 4(\rho_1 + \rho_2)$ and $\rho' = 2\rho_3$. Minimisation with respect to v_L and v_R yields,

$$v_L v_R = \frac{2\left[(\gamma_{11} + \gamma_{22})kk' + \gamma_{12}(k^2 + k'^2)\right]}{\rho - \rho'} \tag{25}$$

This expression simplifies in the limit $k' << k$ to

$$v_L v_R = \frac{2\,\gamma_{12}}{\rho - \rho'}k^2 \tag{26}$$

Here let us introduce the new scalar η which has a *vev* η_0. Now the scalar potential after the SSB will be,

$$V_2 = V_1 - \mu_\eta^2\,\eta_0^2 + \beta_1\,\eta_1\,\eta_0^4 + M\,\eta_0\,(v_L^2 - v_R^2) + \beta_2\,\eta_0^2\,(v_L^2 + v_R^2) + \gamma\,\eta_0^2\,(k^2 + k'^2) \tag{27}$$

Now the minimization with respect to v_L and v_R gives the following relation in the limit $k' << k$,

$$v_L v_R = \frac{2\,\gamma_{12}k^2}{[\rho - \rho' + \frac{4M\eta_0}{(v_L^2 - v_R^2)}]} = \frac{\beta k^2}{[\rho - \rho' + \frac{4M\eta_0}{(v_L^2 - v_R^2)}]} \tag{28}$$

We have defined the new parameter $\beta = 2\gamma_{12}$. At this stage let us introduce the scalar field ξ. This will again introduce new terms in the scalar potential. The scalar potential after SSB now becomes,

$$\begin{aligned}
V_3 &= V_2 + (v_L^2 + v_R^2)\left[(a_{11} + a_{22} + b_{11})\,\tilde{k}^2 + (a_{11} + a_{22} + b_{22})\,\tilde{k}'^2\right.\\
&\quad \left. + (4a_{12} + b_{12})\,\tilde{k}\tilde{k}'\right] + 2v_L v_R[(c_{11} + c_{22})\tilde{k}\tilde{k}' + c_{12}(\tilde{k}^2 + \tilde{k}'^2)]\\
&\quad + \text{terms containing}\ \tilde{k}\ \text{and}\ \tilde{k}'\ \text{only} \tag{29}
\end{aligned}$$

Now we minimize V_3 with respect to v_L and v_R. The see-saw relation becomes,

$$v_L v_R = \frac{\beta k^2 + 2c_{12}\,(\tilde{k}^2 + \tilde{k}'^2)}{[\rho - \rho' + \frac{4M\eta_0}{(v_L^2 - v_R^2)}]} \tag{30}$$

This relation in the limit $\tilde{k}' << \tilde{k}$ becomes,

$$v_L v_R = \frac{\beta k^2 + w\ \tilde{k}^2}{[\rho - \rho' + \frac{4M\eta_0}{(v_L^2 - v_R^2)}]} \tag{31}$$

Here we have defined $w = 2c_{12}$. This is the see-saw condition in the presence of ξ .

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